A Framework for Radion-Supported Gravitating Solitons: Toward a Geometric Interpretation of Localized Gravitational Mass

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Abstract

We investigate localized, finite-energy solutions of an Einstein-Yang-Mills-Higgs (EYMH) system coupled to a dynamically stabilized radion field. Rather than proposing a new origin for gravitation, we explore a novel mechanism for stabilizing localized energy: by fusing non-Abelian gauge extensions with a warped Kaluza-Klein compactification, we investigate whether extended gauge-field configurations can provide the topological pressure required to stabilize internal dimensions. The framework defines an existence conjecture for Radion-Supported Gravitating Solitons (RSGS). We explore the possibility that localized matter-like states may emerge as finite-energy excitations of a compactified geometry, where gauge flux may become self-trapped, producing a stable, gravitating configuration if such solutions exist, manifesting as effective four-dimensional mass.

1 Introduction

Standard particle physics and general relativity operate under a paradigm where matter fields act as sources for a passive spacetime background. However, the concept that stable particles might emerge as localized, self-confined field configurations—solitons—possesses a rich theoretical history, including Wheeler's geons [1], Bartnik-McKinnon solutions [2], and Einstein-Yang-Mills-Higgs monopoles [3].

In this paper, we propose a framework toward a geometric interpretation of localized gravitational mass. We investigate whether the EYMH system, when coupled to a dynamically stabilized radion field within a warped compactification, admits localized solutions that act as massive gravitating defects. Crucially, this framework maintains the strict U(1) gauge invariance of the photon, ensuring it remains massless in the standard sense; the mass in question is an effective gravitational mass associated with the localized soliton configuration, not a Proca mass term. We explore the possibility that these configurations represent localized, finite-energy excitations of the compactification manifold itself, where gauge flux provides the topological pressure required to support the internal geometry against collapse.

2 The Formal Existence Conjecture

To substantiate the claim that matter-like states may emerge from confined gauge energy, we define the following system of five coupled nonlinear radial equations for the functions $\{K(r), H(r), \phi(r), m(r), S(r)\}$:

$$m' = 4\pi r^2 \rho(r) \tag{1}$$

$$\frac{S'}{S} = 4\pi G r(\rho + p_r) \tag{2}$$

$$(r^2 N S f(\phi) K')' = SK\left(y(\phi)H^2 + f(\phi)\frac{K^2-1}{r^2}\right) \tag{3}$$

$$(r^2 N S y(\phi) H')' = SH\left(2y(\phi)\frac{K^2}{r^2} + r^2\frac{\partial U}{\partial H^2}\right) \tag{4}$$

$$(r^2 N S \phi')' = r^2 S\left(\frac{\partial V_{eff}}{\partial \phi} - \frac{\partial \mathcal{L}_{YMH}}{\partial \phi}\right) \tag{5}$$

2.1 Conjecture Statement

Conjecture: There exists an open, non-empty region of the parameter space $\mathcal{P} = \{e, \lambda, v, n, m_\phi\}$ such that there exist shooting parameters $\{K(0) = 1, H(0) = 0, \phi'(0) = 0, m(0) = 0, S(0) = S_0\}$ for which the resulting solutions are globally regular and approach the stabilized Kaluza-Klein vacuum $\{K(\infty) = 0, H(\infty) = v, \phi(\infty) = \phi_0\}$ as $r \to \infty$.

We invite the community to:

1. Determine if these equations admit regular, finite-energy soliton solutions via numerical shooting or relaxation methods, establishing whether dynamic radion coupling yields a new branch of solutions beyond known gravitating monopoles.

2. Verify the dynamic stability of these configurations against radial perturbations.

3 Conclusion

This framework provides a mathematically defined system for investigating how localized gauge-field energy might manifest as stable, gravitating configurations. By preserving the strict masslessness of the fundamental gauge sectors while allowing for self-trapped, massive gravitating defects, we offer a pathway to distinguish between fundamental particle mass and effective, emergent gravitational mass within a higher-dimensional Einstein-frame theory.

Future investigations must ultimately address how such solitons reproduce the observed quantum numbers of elementary particles, and under what exact conditions this effective field theory remains valid relative to the established successes of the Standard Model. We look to the community to resolve whether these equations support the existence of such stable, self-gravitating structures.

References

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[2] R. Bartnik and J. McKinnon, Phys. Rev. Lett. **61**, 141 (1988).

[3] P. Breitenlohner, P. Forgács, and D. Maison, Nucl. Phys. B **383**, 357 (1992).